ADVANECD ALGEBRA, HOMEWORK, 1391.12.01

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Exercise 0.1. *Let* $\mathfrak{a}$ *be an ideal of the commutative ring* R*. Prove that* $\mathfrak{a}$ *is a prime ideal if and only if* $\mathfrak{a}$ *is the kernel of a ring homomorphism from* R *into a field.*

Exercise 0.2. *Let* R *be a commutative ring with* $1 \neq 0$*. Prove that if every proper ideal of* R *is prime, then* R *is a field.*

Exercise 0.3. *Let* R *be a commutative ring. Suppose that* $< x >$ *is a prime ideal of* $R[x]$*. Show that* R *is an integral domain.*

Exercise 0.4. *Show that the ideal* $< x >$ *in* $\mathbb{Z}[x]$ *is a prime ideal, but not a maximal ideal.*

Exercise 0.5. *Let* R *be a commutative ring. Let* $\mathfrak{m}$ *and* $\mathfrak{n}$ *be two distinct maximal ideals of* R*. Show that* $\mathfrak{m} \cap \mathfrak{n} = \mathfrak{m}\mathfrak{n}$.

Exercise 0.6. *A local ring contains no idempotent* $\neq 0, 1$*. (An element* x *in a ring is idempotent if* $x^2 = 1$*.)*

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